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DRAINAGE IN GROUND WATER FLOW OVER A SCREEN

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The paper deals with construction and investigation of a mathematical model for ground water flow from the earth's surface over a vertical impermeable screen with flow interception by a drain located on the screen surface.

Introduction. In [1], we considered the boundary-value problem of steady filtration in a bed of infinite depth and length for ground water flow from the earth's surface flooded everywhere, except for a strip of width $2l$, to a single tubular interceptor (point drain) located at the middle of the strip. In this model, the relationship between the shape of the depression curve and drainage flow rate is established, and the critical drainage regime that arises at the limit of flow destabilization is revealed. Beyond the bounds of the critical regime, the boundary-value problem describes a different, also unstable, filtration process, which keeps only the outward form of the physical matter. Its filling is implemented in the extended formulation of the problem considered below. Possible applications of this problem are calculations of ground water flow interception by horizontal drainage over sheet piles.

Formulation and Solution of the Problem. Ground water coming from the earth's surface flooded at $x \geqslant l$ (Fig. 1) flows over a vertical impermeable screen with vertex G at depth s from the earth's surface and enters an infinite thickness bed. The water filtering from the surface to the right of the screen is acted by backpressure from below, which leads to water flow over the screen. Filtration to the left of the screen occurs only by gravity.

We assume that in this flow regime, a point drain D located on the screen surface at depth $d \geqslant s$ comes into operation and intercepts some amount of water $Q_{\rm dr}$; the rest of water $Q_{\rm f}$ still flows into the depth to the left of the screen. Once the drainage flow rate reaches a certain value $Q_{\rm dr1}$, the drain intercepts the entire flow. Both limiting case — flow over the screen without drainage and total flow interception by drain — were studied in [1]. In Fig. 1, these cases are shown by depression curves 0 and l, respectively; the dashed curve is a streamline that generally separates the flows to the drain and into the depth.

The problem is to find the complex potential $\omega = \varphi + i\psi$ as an analytical function of the complex coordinate $z = x + iy$ of points of the flow region shown in Fig. 1 subject to the boundary conditions

$$BC\colon\quad y = 0,\quad \varphi = 0;\qquad CD\colon\quad x = 0,\quad \psi = 0;$$
$$AD\colon\quad x = 0,\quad \psi = Q_{\rm dr};\qquad AB\colon\quad \varphi - y = 0,\quad \psi = Q, \tag{1}$$

where φ is the filtration velocity potential, ψ is the stream function, and $Q = Q_{\rm dr} + Q_{\rm f}$ is the total filtration flow. The first condition on the depression curve AB follows from equality of pressure on this boundary segment to atmospheric pressure.

Under conditions (1), the regions of the complex potential ω and the analytical Joukowski function $\theta = \omega + iz$ [2] are rectilinear polygons, presented in Figs. 2 and 3. Mapping them conformally onto an auxiliary half-plane $\operatorname{Im}\zeta \geqslant 0$ (Fig. 4), we obtain

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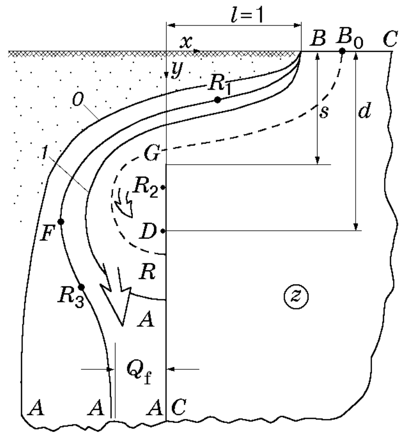

Fig. 1

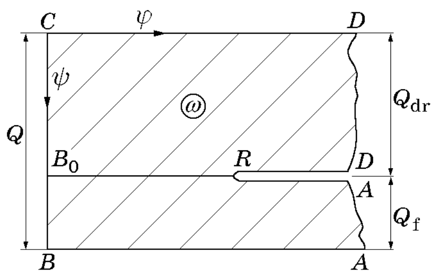

Fig. 2

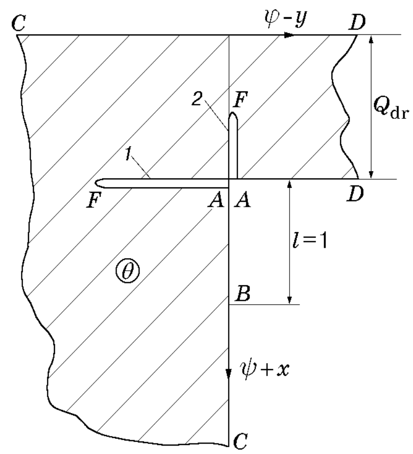

Fig. 3

$$z = \frac{Q_{\rm dr}}{\pi}\Big(\frac{\sqrt{a}}{f}\int\limits_a^\zeta \frac{(u-f)\,du}{u\sqrt{u-a}} + i\,\frac{a}{r}\int\limits_\zeta^1 \frac{(u-r)\,du}{u(u-a)\sqrt{1-u}}\Big) - Q_{\rm f}$$

$$= \frac{2Q_{\rm dr}}{\pi}\Big(\frac{a}{f}\sqrt{\frac{\zeta}{a}-1} - \arctan\sqrt{\frac{\zeta}{a}-1}\Big) - Q_{\rm f} + i\,\frac{2}{\pi}\Big(Q_{\rm dr}\,\mathrm{artanh}\,\sqrt{1-\zeta} + Q_{\rm f}\,\mathrm{artanh}\,\sqrt{\frac{1-\zeta}{1-a}}\Big), \qquad (2)$$

$$\omega = \frac{Q_{\rm dr}}{\pi}\,\frac{a}{r}\int\limits_\zeta^1 \frac{(u-r)\,du}{u(u-a)\sqrt{1-u}} + Q = \frac{2}{\pi}\Big(Q_{\rm dr}\,\mathrm{artanh}\,\sqrt{1-\zeta} + Q_{\rm f}\,\mathrm{artanh}\,\sqrt{\frac{1-\zeta}{1-a}}\Big) + iQ.$$

Below we use normalized quantities z and ω linked to the physical quantities $z_{\rm ph}$ and $\omega_{\rm ph}$ by the equalities

$$z = z_{\rm ph}/l, \qquad \omega = \omega_{\rm ph}/(kl),$$

where k is the soil permeability coefficient.

A goal of the present study of the simulated process is to calculate its hydrodynamic parameters in the direct physical formulation — for specified values of the drain filtration flow $Q_{\rm dr} \in (0, Q_{\rm dr1})$ and the geometrical parameters determining the flow (abscissa $l = 1$ of the boundary of the flooded surface region and the ordinates s and d of the screen vertex G and the drain located on its surface D, respectively). The unknown mapping parameters a, f, and g and the flow $Q_{\rm f}$ not intercepted by the drain are determined from the equations

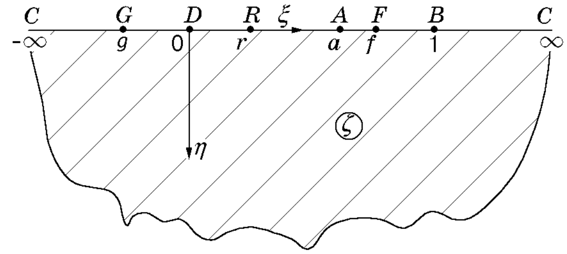

Fig. 4

$$Q_{\rm dr}\Big(\frac{a}{f}\sqrt{\frac{1}{a}-1}-\arctan\sqrt{\frac{1}{a}-1}\Big)=\frac{\pi}{2}(1+Q_{\rm f}),\quad Q_{\rm dr}\Big(\frac{a}{f}-\ln\sqrt{a}\Big)+Q_{\rm f}\,\mathrm{artanh}\sqrt{1-a}=\frac{\pi}{2}\,d,$$

$$Q_{\rm dr}\Big(\frac{a}{f}\sqrt{1-\frac{g}{a}}+\ln\frac{1+\sqrt{1-g}}{\sqrt{a}+\sqrt{a-g}}\Big)+Q_{\rm f}\,\mathrm{artanh}\sqrt{\frac{1-a}{1-g}}=\frac{\pi}{2}\,s, \tag{3}$$

$$f(r-g)\sqrt{a}=r(f-g)\sqrt{(a-g)(1-g)}\qquad (r=aQ_{\rm dr}/(Q_{\rm dr}+Q_{\rm f}\sqrt{1-a})).$$

The last equation of the system follows from the condition that at the screen vertex the filtration velocity tends to infinity. In the plane ζ, the parameter r is the affix of the flow bifurcation point R on the segment AD of the screen surface.

The vertex F of the section along the boundary of the region θ is another moving singular point, whose affix f is also among the unknown mapping parameters for the case of drainage flow operation. The question of the position of this point on the boundary of the flow region is considered below.

Limiting Cases. In the flow considered there are two limiting cases: filtration in the absence of drainage and total flow interception by the drain. Generally, exactly these cases determine the ranges of the required parameters, including drainage flow rate, and, hence, should be calculated first of all.

Let us consider the first case $Q_{\rm dr}=0$. For $Q_{\rm dr}\approx 0$, from the first equation of system (3) and the last equality, for the parameter r, we have

$$f\approx 2Q_{\rm dr}\sqrt{a(1-a)}/[\pi(1+Q_{\rm f})],\qquad r\approx aQ_{\rm dr}/(Q_{\rm f}\sqrt{1-a}\,). \tag{4}$$

With allowance for these relations, in the limit ($Q_{\rm dr}\to 0$), relations (2) are written as

$$z=(1+Q_{\rm f})\sqrt{\frac{\zeta-a}{1-a}}-Q_{\rm f}+iQ_{\rm f}\,\mathrm{artanh}\sqrt{\frac{1-\zeta}{1-a}},\quad \omega=Q_{\rm f}\,\mathrm{artanh}\sqrt{\frac{1-\zeta}{1-a}}+iQ.$$

System (3) becomes

$$(1+Q_{\rm f})\sqrt{\tau(0)}+(2Q_{\rm f}/\pi)\,\mathrm{arcoth}\sqrt{1+\tau(0)}=d,$$

$$(1+Q_{\rm f})\sqrt{\tau(g)}+(2Q_{\rm f}/\pi)\,\mathrm{arcoth}\sqrt{1+\tau(g)}=s, \tag{5}$$

$$\sqrt{\tau(g)(1+\tau(g))}=(2Q_{\rm f}/\pi)/(1+Q_{\rm f})\quad (\tau(\zeta)=(a-\zeta)/(1-a)).$$

The left side of the second equation of system (5) proves to be a function of the quantity $Q_{\rm f}$ after elimination of $\tau(g)$ from it using the third equation. It is established analytically that this function increases monotonically in the interval $(0,\infty)$ with increase of the argument in the same interval, which ensures unique solvability of the second equation of system (5) for $Q_{\rm f}$; in this case, the value of $\tau(g)$ is also determinated uniquely.

The expressions on the left sides of the first two equations of system (5) are values of the ordinate $y(\zeta)$ of the points D and G on the boundary segment AC. In the interval $(-\infty,g)$, the function $y(\zeta)$ decreases, reaching the minimum value s for $\zeta=g$ [this is reflected in the second equation of system (5)]. With further increase of the parameter ζ in the interval (g,a), the function $y(\zeta)$ increases. From this it follows that for $d>s$, the first equation defines two values of the parameter $\tau(0)$: $\tau_{01}\in(0,\tau(g))$ and $\tau_{02}\in(\tau(g),\infty)$. For each of them, from the last equality of system (5), we calculate two pairs of values for the parameters a and g, first of which corresponds to the case of location of the drain on the outer face (relative to the source) of the screen, and the second corresponds to the drain located on the inner face of the screen:

$$a_{01} = \tau_{01}/(1+\tau_{01}), \qquad g_{01} = (\tau_{01} - \tau(g))/(1+\tau_{01}) < 0,$$
$$a_{02} = \tau_{02}/(1+\tau_{02}), \qquad g_{02} = (\tau_{02} - \tau(g))/(1+\tau_{02}) > 0. \tag{6}$$

We note that although in the flow model discussed here, the drain does not operate, its location must be taken into account here because this model serves as the original one for investigation of the flow in which the affix $\zeta = 0$ for the drain D is fixed on the real axis of the plane ζ (see Fig. 4); therefore, the parameters a and g in the case $Q_{\rm dr} = 0$ should be matched to this choice.

In the second limiting case (interception of the entire filtration flow by the drain), the relations for the functions z and ω are obtained directly from formulas (2) for $Q_{\rm f} = 0$. System (3) becomes

$$Q_{\rm dr1}\Big(\frac{a}{f}\sqrt{\frac{1}{a}-1} - \arctan\sqrt{\frac{1}{a}-1}\Big) = \frac{\pi}{2}, \quad Q_{\rm dr1}\Big(\frac{a}{f} - \ln\sqrt{a}\Big) = \frac{\pi}{2}\,d,$$
$$Q_{\rm dr1}\Big(\frac{a}{f}\sqrt{1-\frac{g}{a}} + \ln\frac{1+\sqrt{1-g}}{\sqrt{a}+\sqrt{a-g}}\Big) = \frac{\pi}{2}\,s, \quad g = \frac{1}{2}\Big(1+2f-\sqrt{1+4\,\frac{f^2}{a}\Big(1-\frac{a}{f}\Big)}\Big). \tag{7}$$

The last equality follows from the fourth equation of system (3) for $r = a$.

Besides the mapping parameters a, f, and g, from system (7) we need to determine the flow through the drain $Q_{\rm dr1}$, whose position determines the choice of a computational algorithm.

When the drain D is located on the inner face of the screen, the key calculated characteristic is the ordinate y_A^* of the point A on the depression curve for the critical drainage regime of two-way filtration flow. This flow model, mentioned in the introduction and studied extensively in [1], was the basis for the development of an approach to analyzing multiparameter problems of free-boundary filtration. The value of y_A^* is obtained from the equalities

$$y_A^* = Q_{\rm dr}^* \,\mathrm{artanh}\sqrt{1-a^*}, \qquad Q_{\rm dr}^* = \pi d/(2 - \ln a^*) \tag{8}$$

for the value of the parameter a^* determined from the equation

$$d\big(\sqrt{1/a^*-1} - \arctan\sqrt{1/a^*-1}\,\big) + \ln\sqrt{a^*} - 1 = 0.$$

In the case $s < y_A^*$, the drain intercepts the flow for the drainage flow rate $Q_{\rm dr1} < Q_{\rm dr}^*$ calculated in the process of determining the parameter a_1 from the third equation of system (7) for $a = a_1 = g$:

$$Q_{\rm dr1}\,\mathrm{artanh}\sqrt{1-a_1} = \frac{\pi}{2}\,s, \quad Q_{\rm dr1} = \frac{d\sqrt{1/a_1-1}-1}{\arctan\sqrt{1/a_1-1} - \sqrt{1/a_1-1}\,\ln\sqrt{a_1}}. \tag{9}$$

If $s \geqslant y_A^*$, total flow interception by the drain occurs at the limit of its capabilities, in a critical regime. Furthermore, in the case of the strict inequality, part of the flow intercepted by the drain flows over the screen. This model takes place on the continuation (studied in [1]) of the solution of the initial boundary-value problem over the parameter a to one of the intervals (a^{**}, a^*) or $(0, a^{**})$, depending on whether the drain is on the inner or outer face of the screen. In the corresponding interval, the parameter $a = a_1$ is calculated from the third equation of system (7) from which the parameter f and discharge $Q_{\rm dr1}$ are previously eliminated using the first and second equations and the parameter g is eliminated using the fourth equation. The value of a^{**} is determined from the equation

$$2 - \frac{(1-a^{**})(d\sqrt{1/a^{**}-1}-1)}{d\arctan\sqrt{1/a^{**}-1} - \ln\sqrt{a^{**}}} = 0,$$

which is a consequence of the equality $g = 0$.

General Case: Computational Algorithm and Analysis of the Solution. The key stage in the solution of the primal boundary-value problem involves determination of the parameters a, f, and g and the free filtration flow $Q_{\rm f}$ from system (3) for specified values of the input physical parameters determining the simulated flow and entering the right sides of Eqs. (3). The choice of the drainage flow rate $Q_{\rm dr}$ is limited by its maximum admissible value $Q_{\rm dr1}$, calculated in the limiting regime of total flow interception by the drain. In addition, the ranges (a_1, a_0) and (g_1, g_0) of the required mapping parameters a and g are determined by preliminary computation of both limiting cases.

Generally, system (3) is solved numerically by a two-stage iterative procedure. In the external cycle of the procedure, the parameter $a \in (a_1, a_0)$ is calculated from the third equation of system (3), whose left side can be represented as a complex function of this parameter. In this case, for each value of a, the quantity $Q_{\rm f}$ and the parameters f and g are obtained from the remaining three equations of system (3). In the solution of the fourth equation, one of the two versions — $g \in (0, a)$ or $g \in (-\infty, a)$ — is possible, depending on whether the drain is on the inner or outer faces of the screen, respectively. The monotonicity of the indicated complex function, which ensures unique solvability of the third equation for the parameter a, is established numerically, as in the limiting case of total interception.

Along with the fixed singular points B, D, and G, whose coordinates in the flow plane in the direct formulation are considered specified and are used in derivation of the first three equations of system (3), the moving singular points A, R, and F are also present in the problem. The point A is the end point of the depression curve; determination of the mapping parameter a related to this point constitutes the main part of the computation procedure. The affix r of the flow bifurcation point R on the boundary segment AD changes in the interval $(0, a)$, and $r = 0$ and $r = a$ are for $Q_{\rm dr} = 0$ and $Q_{\rm dr} = Q_{\rm dr1}$ ($Q_{\rm f} = 0$), respectively.

In the model of flow over the screen without drainage, the singular points F and R are absent. Using the relation $\varphi = -p + y$, which links the normalized filtration velocity potential φ and the flow hydrodynamic pressure p normalized by the specific mass of the liquid γ, for the vertical surface of the screen AGC, we obtain

$$\frac{dp}{dy} = 1 - w_y. \tag{10}$$

In the limiting case considered, the vertical filtration velocity component w_y decreases from 0 to $-\infty$ for motion over the inner face of the screen from the point C to the screen vertex G. After transition to the outer face and during subsequent motion down this face, it decreases from ∞ to 1. From this and from equality (10) it follows that along the screen, the flow pressure decreases from the infinitely high value at the point C to atmospheric pressure ($p = 0$) at the point A. We note that for $s = 0$, i.e., in the case of total shielding from the surface source, ground water to the right of the screen, acted upon from below by the upthrust, fills the pores to the earth's surface, on which the source is located. After that, they are in the state of rest, in which the pressure increases with increase in depth under the hydrostatic law; the pressure over the entire depth of the soil to the left of the screen is equal to atmospheric pressure. The filtration flow replenished by the source moves over here from the right with deepening of the screen.

As soon as the drain D on the screen surface comes into operation, a zone of decreased pressure forms in its neighborhood. The boundary of this zone on the segment DA is denoted by the pressure maximum point F; in the region θ, this is the vertex of the boundary section 1 (see Fig. 3). If the drain is located on the inner face of the screen and, in addition, $s < y_A^*$, the drain completely intercepts the flow with certain increase in drainage flow rate within the framework of the two-way flow model [1]; in this case, the point F remains on the inner face of the screen. In the other cases, where there is at least partial flow over the screen, the position and physical meaning of the point F are studied using the last equation of system (3) and the following equality obtained directed from it:

$$f - r = f\,\frac{r - g}{g}\Big(\sqrt{\frac{a}{(a - g)(1 - g)}} - 1\Big). \tag{11}$$

We assume that the drain is located on the inner side of the screen ($0 < g < a$) and the inequality $s > y_A^*$ is satisfied simultaneously. By virtue of the asymptotic relation (4), the parameters f and r are small for small values of $Q_{\rm dr}$: the points F and R are in the immediate neighborhood of the drain. According to (3), we have $r \approx a$ in situations close to drainage with nearly total flow interception, where $Q_{\rm f} \approx 0$. In this case, equality (11) leads to the relation

$$f - a \approx f\,\frac{a - g}{g}\Big(\sqrt{\frac{a}{(a - g)(1 - g)}} - 1\Big) > 0,$$

which implies that with increase in drainage rate, the point F goes over onto the depression curve and becomes its left end point. As a result, the inflection point R_3 appears on the segment AF of the depression curve, in addition to the inflection point R_1, which is originally present on the curve. The indicated change of the filtration flow pattern is associated with section 2 along the boundary of the region θ (see Fig. 3). These transformations, however, do not affect the relation $\theta(\zeta)$, and, hence, the representation of the solution. From the last equation of system (3) and its modification (11) we obtain

$$\operatorname{sign}(f - g) = \operatorname{sign}(r - g) = \operatorname{sign}(f - r). \tag{12}$$

Thus, with increase in drainage flow rate $Q_{\rm dr}$, the point F passes by the screen vertex G simultaneously with the point R, thus moving from the flow intercepted by the drain into the free-stream zone.

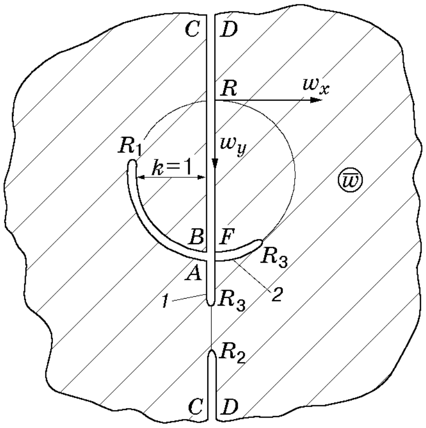

Fig. 5

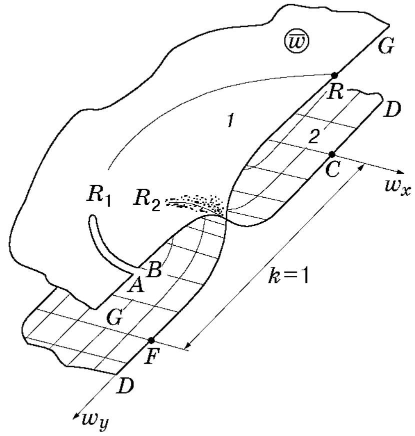

Fig. 6

If the drain is located on the outer face of the screen, the inequalities $-\infty < g < 0$ are satisfied. In view of this and by virtue of relations (11), (12), we have $f > r > g$. This means that when the drain is actuated, the points F and R appear on the same face beneath the drain. Moreover, the point F occurs at once in the free-stream zone, and with further increase in drainage flow rate $Q_{\rm dr}$, it also goes over onto the depression curve.

Velocity Hodograph. In free-boundary filtration problems, the flow pattern it can be judged by a velocity hodograph. An expression for the conjugate complex filtration velocity $w = w_x - iw_y = d\omega/dz$ [2] can be obtained from the integral representations (2) for the functions z and ω. In this case, for points on the screen surface along which $-\infty < \zeta < a$ and $w = -iw_y$, we have

$$\frac{1}{w_y} = 1 - \frac{r}{f\sqrt{a}} \frac{(f-\zeta)\sqrt{(a-\zeta)(1-\zeta)}}{r-\zeta}. \tag{13}$$

In Fig. 5, the hodograph corresponds to flow with operation of a drain located on the outer face of the screen. In this case, in the initial stage of drainage, the points F and R_3 are on the surface of the screen and they are the maximum points of pressure and filtration velocity on the segment FA; the boundary section 1 on the hodograph is directed down along the w_y axis. Using equality (13) and taking into account that $f > r$ and the affix r_3 of the point R_3 is related to the affix r_2 of the vertex of the other section R_2 (velocity minimum points on the segment GD) by the inequality $r_3 > r_2$, we can show that $w_y(R_3) < w_y(R_2)$, and, therefore, these two sections are not overlapped, i.e., the hodograph is one-sheeted. As the drainage flow rate increases, the points F and R_3 go over onto the depression curve (see Fig. 1), section 2 on the hodograph with vertex at the point R_3 is located along the circular arc corresponding to this boundary segment, and the points A and F will change places (see Fig. 5).

The hodograph undergoes more significant changes if the drain operates on the inner face of the screen and $s > y_A^*$. For rather small values of $Q_{\rm dr}$, for which the flows bifurcate on the same side of the screen, the hodograph proves to be two-sheeted with the internal bifurcation point R_2. Its affix r_2 is one of the two complex conjugate solutions of the equation $dw/d\zeta = 0$, for which $\operatorname{Im} r_2 > 0$ (Fig. 6); the third, real root r_1 is the affix of the inflection point R_1 on the depression curve. In the particular case where for a certain value of $Q_{\rm dr}$, the points F and R coincide with the screen vertex G of the dashed sheet 2 of the hodograph degenerates into the point G, at which the final filtration velocity $\bar{w} = iw_y(g)$ is given by the equality

$$1/w_y(g) = 1 - \sqrt{(a-g)(1-g)/a}\,.$$

From this it follows that $|w_y(g)| > 1$ since $g \in (0, a)$. On the single sheet of the hodograph left in this particular case, the infinitely remote point is the point D — an image of the drainage flow. We note that a similar hodograph structure is obtained for free flow over the screen in the absence of drainage; in this case, the point G takes up the position of the point D.

With further increase in $Q_{\rm dr}$, the indicated sheet on the plane $\bar{w}$ is supplemented by the right half-plane, and the hodograph assumes the same structure as in the case of the drain D located on the outer face of the screen, with the same transformation of one of the boundary sections, which involves passage of the point F onto the depression curve; in the case considered, it is only necessary to interchange the positions of the points D and G.

At $s < y_A^*$, total flow interception by the drain operating on the inner face of the screen occurs within the framework of the two-way flow to the drain [1]. In this case, over the entire range of the drainage flow rate $Q_{\rm dr}$, the velocity hodograph remains the same as in Fig. 6, and in the limiting drainage regime, the upper sheet 1 transforms into a half-circle: $|w - i/2| < 1/2$.

Numerical Calculations. Let us illustrate the flow studied here by results of numerical calculations for $s = 0.3$ and $d = 1$. From the second and third equations of system (5), we obtained the value $Q_{\rm f} = 0.1047$ for the filtration flow rate in the absence of drainage. When the drain is located on the inner face of the screen, the critical drainage regime arises, according to (8), for $Q_{\rm dr}^* = 0.6756$ and $y_A^* = 0.8602$ within the framework of the model of two-way flow to the drain, and total flow interception occurs in the normal drainage regime for $Q_{\rm dr} = Q_{\rm dr1} = 0.4016$, calculated from equalities (9); in this case, the point A coincides with the screen vertex G. If the drain at the same depth of location is on the outer face of the screen, it takes up the entire flow over the screen vertex with the filtration flow rate $Q_{\rm dr1}$ nearly identical to the flow rate $Q_{\rm f}$ (the first value exceeds the second only in the seventh decimal digit). The mapping parameters a and g for both limiting cases also differ insignificantly: $a_0 = 3.691 \cdot 10^{-12}$ and $a_1 = 0.923 \cdot 10^{-12}$; $g_0 \approx g_1 \approx -0.0036$. Thus, the drain has an effect on the filtration characteristics of the flow only when it operates on the inner face of the screen. In the other case, the drain is almost completely shielded by the screen from the surface water source, and even at the limit of its capabilities, it does not activate the flow, influencing its structure only in a small neighborhood; we note that in this case, $y_A = 1.0462$ for total flow interception by the drain.

With deepening of the screen, the flow drainage capabilities enhance against the background of general intensification of the flow. In the model considered, the drain discharge reaches the maximum value $Q_{\rm dr1} = Q_{\rm dr}^* = 0.6756$ in the total interception regime with $s = y_A^* = 0.8602$, decreasing as the screen vertex is further lowered to the drain level and is then elevated from the opposite side [1]; in particular, for $s = 0.9$, we have $Q_{\rm dr1} = 0.6736$ and 0.4529 if the drain is located on the inner and outer faces of the screen, respectively. In the first case, of interest is the dynamics of the movable singular points F and R, which was studied above theoretically. For $Q_{\rm dr} = 0.9170 Q_{\rm dr1}$, these points coincide with the screen vertex G, and with further increase in the drainage flow rate, they appear on the outer face of the screen. For $Q_{\rm dr} = 0.9511 Q_{\rm dr1}$, the point F goes over onto the depression curve, bypassing the point A. The latter coincides with the point R in the regime of total flow interception by the drain; in this case, $y_A = 0.9019$.

Conclusions. Within the framework of the multiparameter boundary-value problem of two-dimensional gravity filtration, we considered a hydrodynamic model of drainage in ground water flow over an impermeable screen, which includes as particular or limiting cases some of the models of gravity filtration to a single tubular drain studied in detail in [1][. An approach based on preliminary calculation of limiting flow regimes is used. They determine the limits of possible intensification of drainage, and the ranges of unknown parameters of conformal mappings, calculated from the system of transcendental equations. As a result, based on the direct formulation of the boundary-value problem, a numerical analysis was performed of flow transformations and their associated changes of the velocity hodograph during drainage intensification. The calculations showed that in the case of shielding from the surface source, the effect of drainage on the flow is much weaker.

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